CHAPTER I

INTRODUCTION

The division of one polynomial, $f(x)$, by another polynomial, $g(x)$, to obtain a quotient polynomial, $q(x)$, and a remainder polynomial, $r(x)$, has been studied by many authors. Many useful algorithms for the computation of the quotient and remainder polynomials may be found in [6], and useful convolution matrix be found in [1]. Perhaps one of the most useful applications comes in the representation of integers in binary notation, for which there are numerous analogies with polynomials are defined over commutative rings other than the integers.

In the current paper, the emphasis is not on computational efficiency, but rather on the derivation of explicit algebraic formulae for the remainder and quotient polynomials. It will be shown that the quotient and remainder can be obtained in terms of a recurrent sequence, which allows the coefficients to be progressively and easily computed. The algebraic formula for the remainder has a useful application in the computation of the powers of a square matrix, A, of order m, say. Using the formula, the computation of A^k, is most efficient when k is much greater than m. Although much work has been done recently [3], [5] on simplifying the computation of the powers of a general square matrix, these methods all require the determination of the eigenvalues. The task of determining the eigenvalues of a matrix of high order can, in itself, be a tedious one. The advantage of the method outlined in the present paper does not require a knowledge of the eigenvalues and will work on matrices that may be singular or defective.

We state the problem in Section 3.1 The explicit formulae for the quotient and remainder polynomials are given in Section 3.2 and 3.3 respectively.

This thesis is divided into 4 chapters. Chapter 1 is the introduction. Chapter 2, deals with some preliminaries and gives some useful results that will be used in later chapters. Chapter 3 is the main results of this research. Precisely, we prove in Section 3.1 the generalized quotient theorem. In Section 3.2 we prove the generalized remainder theorem. The conclusion of research is in Chapter 4.