CHAPTER IV

CONCLUSIONS

The quotient and remainder polynomials have been divided where the coefficients are given in terms of a recurrent sequence. Once the divisor is known, the terms of the recurrent sequence can be obtained very quickly. It is conceded that the computation of the coefficients of the quotient and remainder polynomials using the formulae given in Section 3.1 and 3.2 takes longer in terms of CPU time when compared with existing algorithms that can be found in [6]. However, the provision of explicit expressions for these coefficients lends itself to other applications.

One such application is the computation of powers of a square matrix. The gain in computational efficiency is greater when the power required is much greater than the order of the matrix, which is normally the case in practice. The advantage in the method outlined in the previous section lies in the fact that there is no restriction on the matrix itself, which may be singular or defective, or both. In common with the algorithms of [5] and [6], when computing A^n, it is necessary only to compute powers of A directly up to A^{m-1} for a matrix of order m, irrespectively of how large $n-m$ is. However, another advantage of the present method is that it does not require the computation of the eigenvalues or eigenvectors of the given matrix.

We obtain the following results:

1. Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ where $a_n \neq 0$, and $g(x) = x^m - b_{m-1}x^{m-1} - \cdots - b_2x^2 - b_1x - b_0$, be polynomials in $\mathbb{C}[x]$ and suppose that $m = \deg g(x) \leq \deg f(x) = n$, and the quotient on dividing $f(x)$ by $g(x)$ is

$q(x) = d_{n-m}x^{n-m} + d_{n-m-1}x^{n-m-1} + \cdots + d_2x^2 + d_1x + d_0$ then

$$d_{n-m-k} = \sum_{i=0}^{k} s_{k+1-i}a_{n-i}, \text{ for } k = 0, 1, \ldots, n-m$$

where $\{s_r\}$ is the linear recurrent sequence, $s_1 = 1$ and $s_r = \sum_{i=1}^{r-1} b_{m-i}s_{r-i}$, $r = 2, 3, \ldots$.

2. Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ and $g(x) = b_mx^m - b_{m-1}x^{m-1} - \cdots - b_2x^2 - b_1x - b_0$ where $a_n \neq 0$ and $b_m \neq 0$, be polynomials in $\mathbb{C}[x]$ and suppose that $m = \deg g(x) \leq \deg f(x) = n$, and the quotient on dividing $f(x)$ by $g(x)$ is $q(x) = d_{n-m}x^{n-m} + d_{n-m-1}x^{n-m-1} + \cdots + d_2x^2 + d_1x + d_0$ then

$$d_{n-m-k} = \sum_{j=0}^{k} t_{k+1-j}a_{n-j},$$

and

$$q(x) = \sum_{i=0}^{n-m} \left(\sum_{j=0}^{n-m-i} t_{n-m-i+1-j}a_{n-j} \right) x^i.$$

where $\{t_r\}$ is the linear recurrent sequence, $t_1 = \frac{1}{b_m}$ and $t_r = \frac{1}{b_m} \sum_{i=1}^{r-1} b_{m-i}t_{r-i}$ for $r = 2, 3, 4, \ldots$.

3. If $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$, $a_n \neq 0$, then the quotient on dividing $f(x)$ by $x - b$ is

$$q(x) = \sum_{i=0}^{n-1} \sum_{j=1}^{n-1} b^{j-1}a_{i+j}x^i.$$

4. Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ and $g(x) = b_mx^m - b_{m-1}x^{m-1} - \cdots - b_2x^2 - b_1x - b_0$, where $a_n \neq 0$ and $b_m \neq 0$, be polynomials in $\mathbb{C}[x]$ and suppose that $m = \deg g(x) \leq \deg f(x) = n$, then the remainder on dividing $f(x)$ by $g(x)$ is

$$r(x) = \sum_{k=0}^{m-1} \left(a_k + \sum_{i+j=k} b_i \sum_{v=0}^{n-m-j} t_{n-m-j+1-v}a_{n-v} \right) x^k$$

where $\{t_r\}$ is the linear recurrent sequence, $t_1 = \frac{1}{b_m}$, and $t_r = \frac{1}{b_m} \sum_{i=1}^{r-1} b_{m-i} t_{r-i}$ for $r = 2, 3, \ldots$.