CHAPTER V

CONCLUSION

In this thesis,we have the following result.

1. For nonzero $a_0, a_1, a_2 \in \mathbb{C}$, and nonzero tripotent matrices $T_0, T_1, T_2 \in M_n(\mathbb{C})$ satisfying the commutativity property, let A be their linear combination of the form

$$A = a_0T_0 + a_1T_1 + a_2T_2. \tag{3.2.1}$$

The matrix A is tripotent if and only if one of the following conditions holds:

(a) $a_0 = 1, a_1 = -1, a_2 = -1$ or $a_0 = -1, a_1 = -1, a_2 = 1$ and $T_0^2(T_1 - T_2) = T_0(T_1 - T_2)^2 = -T_0^2(T_1 - T_2)$,

(b) $a_0 = 1, a_1 = -2, a_2 = 4$ or $a_0 = -1, a_1 = -2, a_2 = 4$ and $T_0^2(T_1 - 2T_2) = (T_1 - 2T_2) = T_0(T_1 - 2T_2)^2$,

(c) $a_0 = 2, a_1 = -2, a_2 = 1$ or $a_0 = -2, a_1 = -2, a_2 = 1$ and $T_0^2(T_2 - T_1) = T_0 = T_0(T_2 - T_1)^2$,

(d) $a_0 = 1, a_1 = 1, a_2 = 1$ or $a_0 = -1, a_1 = 1, a_2 = 1$ and $T_0^2(T_1 + T_2) = -T_0(T_1 + T_2)^2$,

(e) $a_0 = 1, a_1 = 2, a_2 = 4$ or $a_0 = -1, a_1 = 2, a_2 = 4$ and $T_0^2(T_1 + T_2) = (T_1 + T_2) = -T_0(T_1 + T_2)^2$,

(f) $a_0 = 2, a_1 = 2, a_2 = 1$ or $a_0 = -2, a_1 = 2, a_2 = 1$ and $T_0^2(2T_1 + T_2) = -T_0 = -T_0^2(2T_1 + T_2)$,

(g) $a_0 = \frac{1}{2}, a_1 = \frac{1}{4}, a_2 = \frac{1}{4}$ or $a_0 = -\frac{1}{2}, a_1 = \frac{1}{4}, a_2 = \frac{1}{4}$ and
$T_0^2(\frac{1}{2}T_1 + \frac{1}{2}T_2) = (\frac{1}{2}T_1 + \frac{1}{2}T_2), T_0(\frac{1}{2}T_1 + \frac{1}{2}T_2)^2 = T_0$,
$a_0 = \frac{1}{2}, a_1 = -\frac{1}{4}, a_2 = \frac{1}{4}$ or $a_0 = -\frac{1}{2}, a_1 = -\frac{1}{4}, a_2 = \frac{1}{4}$ and
$T_0^2(\frac{1}{2}T_1 - \frac{1}{2}T_2) = (\frac{1}{2}T_1 - \frac{1}{2}T_2), T_0(\frac{1}{2}T_1 - \frac{1}{2}T_2)^2 = T_0$.

2. Let c_1 and c_2 be nonzero complex number. Let A and B be nonzero complex matrices and $c_1A + c_2B = C$ satisfy $A^3 = A$, $B^s = B$, $AB = BA$, $A \neq B$, and $C^2 = C$. Then $B^2 = B$ or $B^3 = B$.

3. If one of A or B or C is idempotent (or tripotent) and the combination A, B and C is idempotent (or tripotent), then the others matrices must be idempotent (or tripotent).

4. Let $A_1, \ldots, A_n$ be $t_1, \ldots, t_n$-potent respectively. If combination of $A_1, \ldots, A_n$ is idempotent and one of $A_1, \ldots, A_n$ is idempotent (or tripotent) and the combination $A_1, \ldots, A_n$ is idempotent, then $A_1, \ldots, A_n$ each the other matrices must be idempotent (or tripotent).