CHAPTER I

INTRODUCTION

Fixed-point iterations process for nonlinear mappings in Hilbert spaces and Banach spaces including Mann and Ishikawa iterations process have been studied extensively by many authors to approximate fixed point of various classes of operators and to solve variational inequalities in both Hilbert spaces and Banach spaces and the references therein. In 1952, the original Mann iteration was defined in a matrix formulation by Mann[22]. In 1974, Ishikawa[13] introduced the iterative scheme which later, it is said to be Ishikawa iteration and studied its strong convergence theorem for lipschitzian pseudo-contractive mapping in Hilbert spaces. In 1989, Schu[30] introduced Mann iterative scheme and studied the strong convergence theorem for asymptotically nonexpansive mappings in Hilbert spaces, moreover he also studied the strong convergence theorem of Ishikawa iteration for general generation of the concept of asymptotically nonexpansive mappings is that of the asymptotically pseudocontractive mappings. In 1992, Rhoades extended the work of Schu to uniformly convex Banach spaces. In 2000, in spite of the idea of one and two step iterative scheme, Noor [24, 25] introduced a three-step iterative scheme and studied the approximate solution of variational inclusion in Hilbert spaces by using the techniques of updating the solution and the auxiliary principle. Glowinski and Le Tallec [9] used three-step iterative schemes to find the approximate solutions of the elastoviscoplasticity problem, liquid crystal theory, and eigenvalue computation. It has been shown in [9] that the three-step iterative scheme give better numerical results than the two-step and one step approximal iterations. In 1998, Haubruge, Nguyen and Strodiot[11] studied the convergence analysis of three-step schemes of Glowinski and Le Tallec[9] and applied these schemes to obtain new splitting-type algorithms for solving variation inequalities, separable convex programming and minimization of a sum of convex functions. They also proved that three-step iterations lead to highly parallelized algorithms under certain conditions. Thus we conclude that three-step scheme play an important and significant part in solving various problems, which arise in pure and applied sciences. Recently, Xu and Noor [40] introduced and studied a three-step iterative scheme for solving the nonlinear equation $Tx = x$ for asymptotically nonexpansive mappings in Banach space. In 2004, Cho, Zhou and Guo [7] extended the work of Xu and Noor to the three-step iterative scheme with errors and gave weak and strong convergence theorems for asymptotically nonexpansive mappings in a Banach space. Moreover, Suantai [33] extended the theorem of Xu and Noor by gave weak and strong convergence theorems for a new three-step iterative scheme of asymptotically nonexpansive mappings.

This research is organized into 6 chapters as follows. Chapter I is an introduction to the research problem. Chapter II is concerned with some well-known definitions and used useful results that will be used in our main results of this research.

In Chapter III, the modified Noor iterations with errors and new iterations with errors are defined by using an asymptotically nonexpansive self-mapping. Moreover, It is proved that if the mapping is completely continuous, then the modified Noor iterations with errors converges strongly to a fixed point of the mappings. And if the mapping satisfied condition (A) or completely continuous, then the new iterations with errors converges strongly to a fixed point of the

mapping. It also shown that if the our space satisfies Opial' s condition, then we would have wake convergence theorem of our iteration.

In Chapter IV, We establish several strong convergence theorems for the modified Noor iteration with errors for completely continuous asymptotically nonexpansive mappings in the intermediate sense, and weak convergence theorems for asymptotically nonexpansive mappings in the intermediate sense in a uniformly convex Banach space with Opial's condition.

In Chapter V, to construct an iteration scheme for approximating common fixed points of three nonexpansive mappings and three asymptotically nonexpansive mappings to prove some strong and weak convergence theorems for such mappings in a uniformly convex Banach space.

In Chapter VI is the conclusion of this research.