CHAPTER I

INTRODUCTION

The concept of a Γ-semigroup was introduced by Sen [1]. We can see that any semigroup can be considered as a Γ-semigroup. A semigroup theory has developed, and the definition of a semigroup is developed to an ordered semigroup. There are many mathematicians that study an ordered semigroup, and defined some definitions in ordered semigroups which at the same time they are constructed new theorems. By a development of the definition of a semigroup to an ordered semigroup, there are some mathematicians that also developed the definition of a Γ-semigroup to an ordered Γ-semigroup (some authors called po-Γ-semigroup).

In 1964, Clifford and Preston [2] studied the decomposition of a commutative semigroup (without order) into its archimedean components. In 1981, Sen [1] has introduced the concept and notion of a Γ-semigroup. In 1996, Xie and Wu [3] introduced the concepts of extensions of ideals and of n-prime ideals in ordered semigroups, and proved that every $(n-1)$-prime ideal of an ordered semigroup S is an n-prime ideal of S $(n \geq 3)$. They also proved that an ideal I of an ordered semigroup S is prime if and only if any extension of I is $(n-1)$-prime $(n \geq 3)$. In 1997, Kwon and Lee [4] introduced the concepts of weakly prime and weakly semiprime ideals in ordered Γ-semigroups, and gave some characterizations of weakly prime and weakly semiprime ideals in ordered Γ-semigroups analogous to the characterizations of weakly prime and weakly semiprime ideals in ordered semigroups considered by Kehayopulu [5]. In 1997, Xie and Wu [6] proved that the semilattice congruence $\mathcal{N}$ on an ordered semigroup S is the least regular semilattice congruence. In 1998, Kwon and Lee [7] introduced ideals and weakly prime ideals in ordered Γ-semigroups, and gave some characterizations of ideals and weakly prime ideals in ordered Γ-semigroups analogous to the characterizations of ideals and weakly prime ideals in ordered semigroups considered by Kehayopulu [5]. In 1999, Lee and Kwon [8] gave two new characterizations of weakly prime ideals in ordered semigroups. In 1999, Cao [9] introduced the concept of r (l or t)-archimedean ordered semigroups. In 2001, Xie [10] introduced the concept of weakly r-archimedean ordered semigroups, and proved that an ordered semigroup S is a band of weakly r-archimedean ordered subsemigroups of S if and only if S is a regular band of weakly r-archimedean ordered subsemigroups of S and finally, a negative ordered semigroup S is a band of weakly r-archimedean ordered subsemigroups of S if and only if S is a band of r-archimedean ordered subsemigroups of S. In 2006, Kehayopulu and Tsingelis [11] introduced the relation " η ", and they have shown that for commutative ordered semigroups, we have $\eta = \mathcal{N}$. Using the relation η, they proved that the commutative ordered semigroups are, uniquely, complete semilattices of archimedean semigroups. In 2004, Xu and Ma [12] showed that there exists an order-preserving bijection between the set of all prime ideals of an ordered semigroup $(S; \cdot, \leq)$ and the set of all prime ideals of $(S/\mathcal{N}; \cdot, \preceq)$. Moreover, they gave some necessary and sufficient conditions for the natural ordered semigroup $(S/\mathcal{N}; \cdot, \preceq)$ to be a chain. In 2006, Siripitukdet and Iampan [13] characterized the relationship between (ordered) filters, (ordered) s-prime ideals and (ordered) semilattice congruences in ordered Γ-semigroups, and gave some characterizations of (ordered) semilattice congruences on ordered Γ-semigroups. They also proved that for an ordered Γ-semigroup M,

(a) n is the least semilattice congruence on M,

(b) $\mathcal{N}$ is the least ordered semilattice congruence on M,

(c) $\mathcal{N}$ is not the least semilattice congruence in general.

The remainder of the thesis consists of seven chapters: In Chapter II, we give precise definitions, notations and basic results which will be used in the remainder of the thesis. In Chapter III, we introduce the new concept of the extension ($\langle\!\langle A, I\rangle\!\rangle$) $\langle A, I\rangle$ of an (ordered) ideal I by a set A in (ordered) Γ-semigroups, and provide various properties of extensions of (ordered) ideals in (ordered) Γ-semigroups. In Chapter IV, we introduce the new concepts of (ordered) n-prime ideals and (ordered) n-semiprime ideals in (ordered) Γ-semigroups, and characterize the relationship between (ordered) n-prime ideals and (ordered) ideal extensions in (ordered) Γ-semigroups. In Chapter V, we introduce the new concepts of bands of r-archimedean (ordered) Γ-semigroups and bands of weakly r-archimedean (ordered) Γ-semigroups, and find sufficient conditions of a (ordered) Γ-semigroup M so that M is a band of r-archimedean (ordered) Γ-semigroups and bands of weakly r-archimedean (ordered) Γ-semigroups. In Chapter VI, we find sufficient conditions of a commutative (ordered) Γ-semigroup M so that M is decomposable into their archimedean components. In Chapter VII, we introduce the new concept of regular congruences on ordered Γ-semigroups, and find the least regular order on the ordered Γ-semigroup M/ρ with respect to a regular congruence ρ on an ordered Γ-semigroup M. In Chapter VIII, we conclude the results of the thesis.