CHAPTER VIII

CONCLUSIONS

We obtain the following results:

1. Let P be a prime ideal of a Γ-semigroup M and $A \subseteq M$. Then $\langle A, P\rangle$ is a prime ideal of M. Furthermore, $\langle A, \bigcap\limits_{P\in P(M)} P\rangle$ is a semiprime ideal of M if $\bigcap\limits_{P\in P(M)} P \neq \emptyset$.

2. Assume that M is a commutative ordered Γ-semigroup and $A \subseteq M$. If $\{I_i \mid i \in \Lambda\}$ is a collection of ordered prime ideals of M such that $\bigcap\limits_{i\in\Lambda} I_i \neq \emptyset$, then $\langle\langle A, \bigcap\limits_{i\in\Lambda} I_i\rangle\rangle$ is an ordered semiprime ideal of M.

3. If I is an s-prime ideal of a Γ-semigroup M, then $\phi_I = \sigma_I$ and $n \subseteq \phi_I$.

4. If I is an ordered prime ideal of an ordered Γ-semigroup M, then $\Phi_I = (I \times I) \cup (M \setminus I \times M \setminus I)$.

5. Every $(n-1)$-prime ideal of a Γ-semigroup M is an n-prime ideal of M for all integers $n \geq 3$.

6. Every ordered $(n-1)$-prime ideal of M is an ordered n-prime ideal of an ordered Γ-semigroup M for all integers $n \geq 3$.

7. An ideal I of a commutative Γ-semigroup M is an n-prime ideal of M if and only if any extension of I is an $(n-1)$-prime ideal of M for all integers $n \geq 3$.

8. An ordered ideal I of a commutative ordered Γ-semigroup M is an ordered n-prime ideal of M if and only if any extension of I is an ordered $(n-1)$-prime ideal of M for all integers $n \geq 3$.

9. A Γ-semigroup M is a band of weakly r-archimedean sub-Γ-semigroups of M if and only if it satisfies the condition for all $a, x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$,

$$(a, a\gamma a), (x\alpha a, x\alpha a\gamma a), (a\beta y, a\gamma a\beta y), (x\alpha a\beta y, x\alpha a\gamma a\beta y) \in \eta_r \cap \eta_r^{-1}. \quad (\star)$$

10. An ordered Γ-semigroup M is a band of weakly r-archimedean sub-Γ-semigroups of M if and only if it satisfies the condition for all $a, x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$,

$$(a, a\gamma a), (x\alpha a, x\alpha a\gamma a), (a\beta y, a\gamma a\beta y), (x\alpha a\beta y, x\alpha a\gamma a\beta y) \in \bar{\eta}_r \cap \bar{\eta}_r^{-1}. \quad (\star)$$

11. Let

$$\begin{aligned}\rho \;:=\; & \{(a,b) \mid (a,b), (x\alpha a, x\alpha b), (a\beta y, b\beta y), (x\alpha a\beta y, x\alpha b\beta y) \in \eta_r \cap \eta_r^{-1} \\ & \text{for all } x, y \in M \text{ and } \alpha, \beta \in \Gamma\}\end{aligned}$$

be a congruence on a Γ-semigroup M. Then M/ρ is an ordered Γ-semigroup.

12. Let

$$\begin{aligned}\rho \;:=\; & \{(a,b) \mid (a,b), (x\alpha a, x\alpha b), (a\beta y, b\beta y), (x\alpha a\beta y, x\alpha b\beta y) \in \bar{\eta}_r \cap \bar{\eta}_r^{-1} \\ & \text{for all } x, y \in M \text{ and } \alpha, \beta \in \Gamma\}\end{aligned}$$

be a congruence on an ordered Γ-semigroup M. Then ρ is a regular congruence on M.

13. Let M be a Γ-semigroup. Then following statements are equivalent:

    (a) M is a band of weakly r-archimedean left ideals of M.

    (b) M is a band of r-archimedean left ideals of M.

    (c) M satisfies the condition for all $a, x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$,

$$(a, a\gamma a), (x\alpha a, x\alpha a\gamma a), (a\beta y, a\gamma a\beta y), (x\alpha a\beta y, x\alpha a\gamma a\beta y) \in \eta_r \cap \eta_r^{-1}. \quad (\star)$$

14. Let M be a negative ordered Γ-semigroup. Then the following statements are equivalent:

    (a) M is a band of weakly r-archimedean sub-Γ-semigroups of M.

    (b) M is a band of r-archimedean sub-Γ-semigroups of M.

    (c) M satisfies the condition for all $a, x, y \in M$ and $\alpha, \beta, \gamma \in \Gamma$,

$$(a, a\gamma a), (x\alpha a, x\alpha a\gamma a), (a\beta y, a\gamma a\beta y), (x\alpha a\beta y, x\alpha a\gamma a\beta y) \in \overline{\eta}_r \cap \overline{\eta}_r^{-1}. \quad (\star)$$

15. If M is a commutative Γ-semigroup, then M is a semilattice of archimedean sub-Γ-semigroups of M.

16. If M is a commutative ordered Γ-semigroup, then M is an ordered semilattice of archimedean sub-Γ-semigroups of M.

17. If M is a commutative Γ-semigroup, then M is, uniquely, a semilattice of archimedean sub-Γ-semigroups of M.

18. If M is a commutative ordered Γ-semigroup, then M is, uniquely, an ordered semilattice of archimedean sub-Γ-semigroups of M.

19. Let ρ be a regular congruence on an ordered Γ-semigroup M. Define a relation $(\leq \circ\rho)/\rho$ on M/ρ as follows:

$$(\leq \circ\rho)/\rho \ := \ \{((x)_\rho, (y)_\rho) \mid (x_1, y_1) \in (\leq \circ\rho) \text{ for some } x_1 \in (x)_\rho \text{ and } y_1 \in (y)_\rho\}.$$

    If $\preceq := \{((x)_\rho, (y)_\rho) \mid ((x)_\rho, (y)_\rho) \in ((\leq \circ\rho)/\rho)^m \text{ for some } m \in \mathbb{N}\}$, then $\preceq$ is the least regular order on M/ρ with respect to the regular congruence ρ on M.

20. If $\rho \in SC(M)$, then $(M/f(\rho); \preceq')$ is an ordered Γ-semigroup.

21. If $\rho \in OSC(M)$, then $(M/F(\rho); \preceq')$ is an ordered Γ-semigroup. Moreover, if $x \leq y$, then $F(x)_\rho \preceq' F(y)_\rho$.

22. If $\rho \in SC(M)$, then $M/\rho \cong M/f(\rho)$.

23. If $\rho \in OSC(M)$, then $M/\rho \cong M/F(\rho)$. Moreover, if $x \leq y$, then $F(x)_\rho \preceq' F(y)_\rho$.