

CHAPTER I

INTRODUCTION

In 1965, Kirk [1] proved that if C is a weakly compact convex subset of a Banach space with normal structure, then every nonexpansive self-mapping T of C has a fixed point. A nonempty convex subset C of a normed linear space is said to have normal structure if each convex subset of C consisting of more than one point contains a nondiametral point. That is, a point $x \in K$ such that $\sup\{\|x - y\| : y \in K\} < \sup\{\|u - v\| : u, v \in K\} = \text{diam } K$. Seven years later, in 1972, Goebel and Kirk [2] proved that if the space X is assumed to be uniformly convex, then every asymptotically nonexpansive self-mapping T of C has a fixed point. This was extended to mappings of asymptotically nonexpansive type by Kirk in [3]. More recently these results have been extended to wider classes of space, see for examples [2, 4, 5, 6, 7, 8]. In particular, Lin, Tan and Xu [6] have demonstrated the existence of fixed points for asymptotically nonexpansive mappings in Banach space with uniform Opial condition. It is known [9] that if the Maluta's constant $D(X) < 1$, then X is reflexive and has normal structure and hence the fixed point property for nonexpansive mappings. However, it is not clear if $D(X) < 1$ implies the fixed point property for asymptotically nonexpansive mappings. In 1994, Lim and Xu [10] proved two partial answers to this question.

In this paper, we present some fixed points of asymptotically nonexpansive type mappings.

This thesis is divided into 4 Chapter. Chapter 1 is the introduction. Chapter 2, deals with some preliminaries and give some useful results that will be in later chapter. Chapter 3 is the main results of this research. Precisely, we prove in section 3.1 the demiclosedness principle at zero of asymptotically nonexpansive type mappings. In section 3.2 we prove three fixed point theorems

for asymptotically nonexpansive type mappings. Section 3.3 we prove three weak convergence theorems for asymptotically nonexpansive type mappings. Finally, we also prove in section 3.3 the strong convergence for asymptotically nonexpansive mappings. The conclusion of research is in Chapter 4.

