CHAPTER IV

CONCLUSION

Let X be a Banach space C is nonempty compact convex subset of X let $T : C \to C$ be a asymptotically nonexpansive type, we obtain the following results:

1. X satisfying Opial's condition $T : C \to C$ is a uniformly continuous mapping of asymptotically nonexpansive type. Suppose also $\{x_n\}$ is a sequence in C converges weakly to x and for which the sequence $\{x_n - Tx_n\}$ converges strongly to 0. Then $\{T^n x\}$ converges weakly to x.

2. X satisfying the locally uniform Opial condition, C is nonempty weakly compact convex subset of X and $T : C \to C$ is uniformly continuous mapping of asymptotically nonexpansive type. Then $I - T$ is demiclosed at zero.

3. X is a Banach space satisfying the locally uniform Opial condition, C is nonempty weakly compact convex subset of X and $T : C \to C$ is an asymptotically nonexpansive mapping. Then $I - T$ is demiclosed at zero.

4. X satisfying Opial's condition and whose norm is UKK and C is nonempty weakly compact convex subset of X, and $T : C \to C$ is a uniformly continuous mapping of asymptotically nonexpansive type. Then $I-T$ is demiclosed at zero.

5. X is a Banach space satisfying Opial's condition and whose norm is UKK and C is nonempty weakly compact convex subset of X, and $T : C \to C$ is an asymptotically nonexpansive mapping. Then $I - T$ is demiclosed at zero.

6. X is a Banach space such that $D(X) < 1$, that K is closed bounded convex subset of X, and that $T : K \to K$ is an asymptotically nonexpansive type mapping and weakly asymptotically regular on C. If $\{T^{n_k} x\}$ is a subsequence of $\{T^n x\}$ converges weakly to $x \in K$, then $\limsup_{k\to\infty} \|T^{n_k} x - x\| = 0$.

7. X is a Banach space such that $D(X) < 1$, that C is a nonempty closed bounded convex subset of X, and $T : C \to C$ is continuous of asymptotically nonexpansive type mapping and T is weakly asymptotically regular on C. Further, suppose that there exists a nonempty closed convex subset K of C with the following property (ω):

$$x \in K \text{ implies } \quad \omega_w(x) \subset K,$$

where $\omega_w(x)$ is the weak ω-limit set of T at x; that is, the set

$$\{y \in X : y = weak - \lim_i T^{n_i}x \text{ for some } n_i \uparrow \infty\}.$$

Then T has a fixed point in K.

8. X is a Banach space such that $D(X) < 1$ let $T : C \to C$ be an asymptotically nonexpansive mappings. Suppose there exists a nonempty bounded closed convex subset of K of C with the property(ω). Then T has a fixed point in K.

9. X is a Banach space such that $D(X) < 1$, let C be a bounded closed convex subset of X, and suppose $T : C \to C$ is a continuous mappings of asymptotically nonexpansive type. Then T has a fixed point.

10. X is a Banach space which is uniformly convex in every direction and for which $D(X) < 1$ and that C is a closed bounded convex subset of X. Then, if $T : C \to C$ is a continuous mappings of asymptotically nonexpansive type, T has a fixed point.

11. X is a Banach space with a weakly continuous duality map J_φ, C is a weakly compact convex subset of X, and $T : C \to C$, is an asymptotically nonexpansive type. Furthermore, suppose that there exists a nonempty closed convex subset K of C with the following property (ω):

$$x \in K \text{ implies } \omega_w(x) \subseteq K$$

where $\omega_w(x)$ is the weak $\omega - limit$ set of T at x; that is, the set

$$\{y \in K : y = weak - \lim_i T^{n_i}x \text{ for some } n_i \uparrow \infty\}.$$

Then T has a fixed point in K.

12. X is a Banach space with a weakly continuous duality map J_φ, C is a weakly compact convex subset of X, and $T : C \to C$, is an asymptotically nonexpansive type. Then T has a fixed point in C.

13. X is a Banach space with a weakly continuous duality map J_φ, C is a weakly compact convex subset of X, and $T : C \to C$, is an asymptotically nonexpansive mapping. Further, suppose that there exists a nonempty closed convex subset K of C with the following property (ω):

$$x \in K \text{ implies } \omega_w(x) \subseteq K$$

where $\omega_w(x)$ is the weak $\omega - limit$ set of T at x; that is, the set

$$\{y \in K : y = weak - \lim_i T^{n_i}x \text{ for some } n_i \uparrow \infty\}.$$

Then T has a fixed point in K.

14. X satisfying Opial's condition and let T be a mapping of asymptotically nonexpansive type on C. Let $\{x_n\}$ be a sequence in C which satisfies the following condition

$$w - \lim_n T^m x_n = z_m, \quad \text{for all } m \geq 0.$$

Then $\lim_{m\to\infty} b_m = \inf\{b_m : m \geq 0\}$, where $b_m = \limsup_{n\to\infty} \|T^m x_n - z_m\|$.

15. X is a Banach space satisfying the uniform Opial condition, C is a nonempty weakly compact convex subset of X, and $T : C \to C$ is continuous of asymptotically nonexpansive type mapping. Then T has a fixed point.

16. X is a Banach space satisfying the uniform Opial condition, C is a nonempty weakly compact convex subset of X, and $T : C \to C$ is an asymptotically nonexpansive mapping. Then T has a fixed point.

17. X is a Banach space satisfying the uniform Opial condition, C is nonempty weakly compact convex subset of X and $T : C \to C$ is an asymptotically nonexpansive type. Then given an $x \in C$, $\{T^n x\}$ converges weakly to a fixed point of T if and only if T is weakly asymptotically regular at x.

18. X is a Banach space satisfying the uniform Opial condition, C is nonempty weakly compact convex subset of X and $T : C \to C$ is an asymptotically nonexpansive mapping. Then given an $x \in C$, $\{T^n x\}$ converges weakly to a fixed point of T if and only if T is weakly asymptotically regular at x.

19. X is a Banach space with a weakly continuous duality map J_φ, C is a weakly compact convex subset of X, and $T : C \to C$, is an asymptotically nonexpansive type. Then if T is weakly asymptotically regular at $x \in C$, then $\{T^n x\}$ converges weakly to a fixed point of T.

20. X is a Banach space with a weakly continuous duality map J_φ, C is a weakly compact convex subset of X, and $T : C \to C$, is an asymptotically nonexpansive mapping. Then if T is weakly asymptotically regular at $x \in C$, then $\{T^n x\}$ converges weakly to a fixed point of T.

21. X is a reflexive Banach space with weakly continuous duality map and uniformly Gâteaux differentiable norm. Suppose in addition T are weakly asymptotically regular and completely continuous. Then $\{x_n\}$ converges strongly to a fixed point of T.

22. X be a Banach space with a uniformly Gâteaux differentiable norm such that $D(X) < 1$, that C is a closed bounded convex subset of X, and that $T : C \to C$ is an asymptotically nonexpansive mapping. Then, a mapping S_n on C given by (3.3.1) has a unique fixed point x_n in C. Further, if T is weakly asymptotically regular and completely continuous, then $\{x_n\}$ define by (3.3.2) converges strongly to a fixed point of T.

23. X satisfying Opial's condition and whose norm is UKK. Let C be weakly compact convex subset of X and let $T : C \to C$ be an uniformly continuous of asymptotically nonexpansive type and which is asymptotically regular at the point $x \in C$. Then the iterates $\{T^n x\}$ converges weakly to a fixed point of T.

24. X satisfying Opial's condition and whose norm is UKK. Let C be weakly compact convex subset of X and let $T : C \to C$ be an asymptotically nonexpansive mapping such that T^N continuous for some $N \geq 1$ and which is asymptotically regular at the point $x \in C$. Then the iterates $\{T^n x\}$ converges weakly to a fixed point of T.