CHAPTER IV

CONCLUSIONS

The following is the list of the main results of this study.

1. Let (X,d) be a complete convex metric space, K be a nonempty closed subset of X. Let $S,T : K \to X$ be a non-self mapping such that $T(\partial K) \cup S(\partial K) \subseteq K$ and $\phi : [0,\infty) \to [0,\infty)$ satisfy the following:

   (i) ϕ is continuous and strictly increasing in $\mathbb{R}^+$;

   (ii) $\phi(t) = 0$ if and only if $t = 0$, and if a, b and c are three decreasing functions from $\mathbb{R}^+ \cup \{0\}$ into $[0,1)$ such that $a(t) + 2b(t) + c(t) < 1$, for all $t > 0$. Suppose that S and T satisfies the following condition

$$\begin{aligned}\phi\left[d(Sx,Ty)\right] \;\leq\; & a(d(x,y))\phi(d(x,y)) + b(d(x,y))\left[\phi(d(x,Sx) + \phi(d(y,Ty))\right] \\ & + c(d(x,y))\min\left\{\phi(d(x,Ty)), \phi(d(y,Sx)\right\}\end{aligned}$$

Then S and T have unique common fixed point.

2. Let $T : K \to X$ be a non-self mapping satisfying $T(\partial K) \subseteq K$ of a complete convex metric space (X,d) and $\phi \in \Phi$ such that for every $x, y \in \partial K$,

$$\begin{aligned}\phi\left[d(Tx,Ty)\right] \;\leq\; & a\phi(d(x,y)) + b\phi(d(x,Tx)) \\ & + c\phi(d(y,Ty))\end{aligned}$$

where a, b and c are three nonnegative constants satisfying $a + b + c < 1$. Then T has a unique fixed point.

3. Let X be a uniformly convex Banach space and $\overline{B_r} = \{x \in X : \|x\| \leq r\}$ with $r > 0$. Suppose $F : \overline{B_r} \to X$ is nonexpansive such that $x = \lambda F(x)$ for all $x \in \partial\overline{B_r}$ and for all $\lambda \in (0,1)$. Then F has a fixed point in $\overline{B_r}$.

4. Let K be a nonempty closed bounded convex subset of a uniformly convex in every direction (UCED) Banach space X and $T : K \to X$ be nonexpansive satisfying one of the following:

(i) $T(\partial K) \subseteq K$,

(ii) T is weakly inward condition,

(iii) $0 \in IntK$ and $Tx \neq mx$ for all $x \in \partial K$ and $m > 1$ holds. Suppose for some $u \in K$ and $n \geq 1$. Then T has a unique fixed point.

5. Let X be a reflexive Banach space and let C be a nonempty bounded closed convex subset of X which has normal structure. Let $T : C \to X$ be nonexpansive mapping satisfying $0 \in IntC$ and $Tx \neq mx$ for all $x \in \partial C$ and $m > 1$ or Leray-Schauder's condition. Then T has a fixed point.

6. Let X be a reflexive Banach space and let C be a nonempty bounded closed convex subset of X which has normal structure. Let $T : C \to X$ be nonexpansive mapping satisfying weakly inward condition. Then T has a fixed point.

7. Let X be a reflexive Banach space and let C be a nonempty bounded closed convex subset of X which has normal structure. Let $T : C \to X$ be nonexpansive mapping satisfying inward condition. Then T has a fixed point.

8. Let X be a reflexive Banach space and let C be a nonempty bounded closed convex subset of X which has normal structure. Let $T : C \to X$ be nonexpansive mapping satisfying Rothe's condition. Then T has a fixed point.

9. Let K be open subset of a Banach space X and let $T : K \to X$ be a h_λ-contractive for $h > 0$ and $\lambda \in (0,1)$. Then $(I - T)(K)$ is an open subset of X.

10. Let K be a nonempty closed bounded convex subset of a Banach space X with $IntK \neq \emptyset$ and let $T : K \to X$ be h-nonexpansive which satisfies Rothe's condition. Then for $\lambda \in (0,1)$ sufficiently small, the mapping $T_\lambda : K \to X$ defined by

$$T_\lambda(x) = (1-\lambda)x + \lambda Tx, \quad x \in K,$$

then there exists $z \in K$ such that $\|z - Tz\| \leq h$ for $h > 0$
(i.e., $\inf \{\|x - Tx\| : x \in K\} \leq h$).

11. Let K be a nonempty closed bounded convex subset of a Banach space X with $intK \neq \emptyset$ and let $T : K \to X$ be nonexpansive which satisfies Rothe's condition. Then there exists an approximate fixed point sequence $\{x_n\}$ in K. .