CHAPTER I

INTRODUCTION

The study of fixed point theorems of multi-valued mappings started from von Neumann [1] in case of continuous mappings to multi-valued mappings. Since then, various notion of the fixed point theorems for the multi-valued mappings have been studied in [2, 3, 4]. Recently, the fixed point theorems for multi-valued mapping were generalized and improved by many authors: see, for example, [5, 6, 7, 8, 9, 10].

In 1994, the equilibrium problems were introduced by Blum and Oettli [11] and by Noor and Oettli [12] as generalizations of variational inequalities and optimization problems. The equilibrium problem theory provides a novel and united treatment of a wide class of problems which arise in economics, finance, image reconstruction, ecology, transportation, elasticity, optimization and network. This theory has had a great impact and influence in the development of several branches of pure and applied sciences. Classical examples of equilibrium problems are variational inequalities, optimization problems, complementarity problems. Presently, many results on existence of solutions for vector variational inequalities (in short, VVI) and vector equilibrium problems (in short, VEP) have been established (see, for example, [13, 14, 15, 16, 17, 18, 19]).

In 2000, Ansari, et al. [20] introduced the system of vector equilibrium problems (for short, $SVEP$), that is, a family of equilibrium problems for vector valued bifunctions defined on a product set, with applications in vector optimization problems and Nash equilibrium problem [21] for vector valued functions. The $SVEP$ contains system of equilibrium problems, systems of vector variational inequalities, system of vector variational-like inequalities, system of optimization problems and the Nash equilibrium problems for vector valued functions as special cases.

In recent years, the vector equilibrium problems have been intensively studied by many authors, which is a unified model of several problems, for example, vector optimization problem, vector variational inequality problem, vector complementarity problem and vector saddle point problem. For example, in 2003, the generalized vector quasi-equilibrium problem (for short, $GVQEP$) was introduced by Ansari, et al.[22] which is general form of $SVEP$ and the vector quasi-equilibrium problems was generalized to the system case by Ansari, et al. [23].

On the other hand, a very important result is well known that Ekeland's variational principle (for short, EVP) was first presented by Ekeland [24] in 1974. It is a powerful tool on many applications in optimization, nonlinear analysis, mathematical economy and mathematical programming. Moreover, EVP is equivalent to the Caristis fixed point theorem [25, 26] and nonconvex minimization theorem according to Takahashi [27]. The studies of several forms of Ekeland's variational principle for vector valued functions were obtained by many authors, for instance, Nemeth [28], Tammer [29] and Isac [30, 31].

By the way, determining the stationary points (maximum, minimum and saddle points) of energy surfaces is important in chemical physics because they indicate the equilibrium geometries and transition states which can be used to describe reaction dynamic by classical equation using these stationary points. In n-dimensional real space, the saddle point of order m is the point which is maximum point in directions of m degree of freedoms and minimum point in directions of n-m degree of freedoms [32]. However, the saddle point generally defined and used in literatures is the saddle point of order one which is a maximum point in a direction and minimum point in the other directions. An example of utilising saddle point order one is calculation of reaction rate in chemical physics by harmonic transition state theory (hTST) equation [33, 34], which requires the saddle point order one on its general formula. Studies on saddle points of scalar functions have been generalized to study of saddle points, with respect to a cone, of vector valued functions under necessary and sufficient conditions; see, for example,[35, 36, 37, 38, 39, 40].

Motivated and inspired by the above works, the purposes of this dissertation are to extend, to generalize and to improve existence theorems for finding the solutions of vector equilibrium problems, variational inequality problems, fixed point problems and saddle point problems in topological vector spaces and G-convex spaces.

This dissertation is organized into 5 chapters. Chapter I is an introduction to the dissertation problems. Chapter II is concerned with some notations, definitions, and some useful results that will be used in our main results of this dissertation.

Chapter III and IV are the main results of this research. In the first part, section 3.1, we establish an existence theorem of strong solution set for the system of generalized strong vector quasi-equilibrium problem by using Kakutani-Fan-Glicksberg fixed point theorem and discuss the closedness of the solution set. In section 3.2, we prove the existence theorems of the generalized strong vector quasi-equilibrium problems in locally G-convex spaces, by using Kakutani-Fan-Glicksberg fixed point theorem for upper semicontinuous set-valued mapping with nonempty closed acyclic values, and the closedness of $V_S(F)$ and $V_W(F)$.

In section 3.3, the sufficient conditions are given for the existence of $x \in K$ such that $F(T) \cap VEP(F) \neq \emptyset$, where K is a nonempty compact convex subset of a topological vector space, $F(T)$ is the set of all fixed points of the multi-valued mapping T and $VEP(F)$ is the set of all solutions for vector equilibrium problem of the vector valued mapping F. Moreover, we also study the existence solutions of intersection between the set of all fixed points of the multi-valued mapping and the set of all solutions for vector variational. Section 3.4 deals with the new type of generalized strong vector quasi-equilibrium problems in topological vector spaces. Using the generalization of Fan-Browder fixed point theorem, we provide existence theorems for the new type of generalized strong vector quasi-equilibrium problems with and without monotonicity.

In section 4.1, we establish a vectorial form of Ekeland-type variational principle for multi-valued bioperator whose domain is a complete metric space and its range is a subset of a locally convex Hausdorff topological space by using the set theoretic methods. We also consider Caristi-Kirk fixed point theorem in a more general setting.

In section 4.2, we introduce the n-vectorial saddle point problem (for short VSP_n) which defined on n-dimensional saddle point where n > 2 by focusing only on the saddle point of order one. For that matter, we prove existence saddle point of (VSP_n) under assuming compactness and uncompactness by using Fan-KKM theorem. Finally, we summarize the main results and put some open problems in chapter V.