CHAPTER I

INTRODUCTION

Variational inequality theory with applications is an important part of nonlinear analysis. It has been applied intensively to different fields such as mechanics, game theory, economics, optimization theory and nonlinear programming. Since 1960s, researchers have obtained many existence results of solutions for variational inequality problems and nonlinear complementarity problems, see [1, 2, 3, 4, 5, 6, 7, 8, 9]. Moreover, variational inequality theory has become a very effective and powerful tool for studying a wide range of problems arising in pure and applied sciences which include work on differential equations, control problems, mechanics, general equilibrium problems in transportation and economics. In 1994, Hassouni and Moudafi [10] introduced and studied the class of variational inclusions and developed a perturbed algorithm for finding approximate solutions of the variational inclusions. In 1996, Adly [11] obtained some important extensions and generalizations of the results of Hassouni and Moudafi [10] for nonlinear variational inclusions. In recent years, many authors have been extended and generalized in several directions, using new and powerful methods, to study a wide class of unrelated problems in a unified and general framework, for example, generalized variational inequality problems, system of variational inequality problems, etc. In 2009, Zou and Huang [12] introduced and studied a new class of system of variational inclusions involving $H(\cdot,\cdot)$-accretive operator in Banach spaces. By using the resolvent operator technique associated with $H(\cdot,\cdot)$-accretive operator, they proved the existence of the solution for the system of inclusions. Moreover, they also develop a step-controlled iterative algorithm to approach the unique solution. Recently, Chang, et al. [13] introduced a system of generalized nonlinear variational inequality and an iterative scheme for finding a solution to a system of generalized nonlinear variational inequality problem by using the generalized projection method. Moreover, they

proved some existence and strong convergence theorems in uniformly smooth and strictly convex Banach spaces.

On the other hand, the equilibrium problem is a one important topics of mathematical sciences such as optimization problems, problems of Nash equilibrium, variational inequality problems, complementary problems, fixed point problems; it unifies the above problems in a very convenient way. In 1994, the equilibrium problems were introduced by Blum and Oettli [14] and by Noor and Oettli [15] as generalizations of variational inequalities and optimization problems. The equilibrium problem theory provides a novel and united treatment of a wide class of problems which arise in finance, economics, ecology, image reconstruction, network, transportation, elasticity and optimization. This theory has had a great impact and influence in the development of several branches of pure and applied sciences.

Motivated and inspired by the above works, the purposes of this dissertation are to extend, to generalize and to improve existence theorems and a new iterative schemes for finding the solutions of variational inequality problems, variational inclusions problems and equilibrium problems in Hilbert spaces, Banach spaces and other topological spaces.

This dissertation is organized into 4 chapters. Chapter I is an introduction to the dissertation problems. Chapter II is concerned with some notations, definitions, and some useful results that will be used in our main results of this dissertation.

Chapter III is the main results of this research. In the first part, section 3.1, we introduce a new generalized system of nonlinear variational inequality problem $(GSNVIP)$ by using the generalized projection method. Moreover, we introduce an iterative scheme for finding a solution to this problem. Moreover, some existence and strong convergence theorems are established in uniformly smooth and strictly convex Banach space under suitable conditions.

In section 3.2, we prove the existence of solutions of generalized variational inequality for upper semicontinuous multi-valued mappings with compact contractible values over compact convex subsets in a reflexive Banach space with a Fréchet differentiable norm. Moreover, we give some conditions that guarantee the existence of solutions of generalized variational inequality for upper semicontinuous multi-valued mappings with compact contractible values over unbounded closed convex subsets.

In section 3.3, we study a new system of nonlinear set-valued variational inclusions involving a finite family of $H(\cdot,\cdot)$-accretive operators in Banach spaces. By using the resolvent operator technique associated with a finite family of $H(\cdot,\cdot)$-accretive operators, we prove the existence of the solution for the system of nonlinear set-valued variational inclusions. Moreover, we introduce a new iterative scheme and prove a strong convergence theorem for finding a solution of this system.

In section 3.4, we introduce a new generalized mixed equilibrium problem with a relaxed monotone mapping. By using KKM theorem, we establish an existence theorem for this problem in a Hausdorff topological vector space. Moreover, we introduce an iterative sequence and prove a weak convergence theorem for a generalized mixed equilibrium problem with a relaxed monotone mapping in Hilbert space. Finally, we summarize the main results in chapter IV.