CHAPTER IV

CONCLUSION

In this thesis, we have the following results:

1. Let $\rho \in S_n$ be a cycle of even length. Then $\det(I + A_\rho) = 0$ where A_ρ is permutation matrix corresponded to ρ.

2. Let $\rho \in S_n$ be a cycle of odd length. Then $\det(I + A_\rho) \neq 0$ where A_ρ is permutation a matrix corresponded to ρ.

3. Let $\sigma = \rho_1\rho_2 \ldots \rho_t \in S_n$ where $1 \leq t \leq n$ and ρ_ℓ, $1 \leq \ell \leq t$ is a cycle of even length then, $\det(I + A_\sigma) = 0$, where A_σ the permutation matrix corresponded to σ.

4. If A and B are permutation matrices in $P(n)$ and $A^{-1}B$ corresponding to permutation $\sigma = \rho_1\rho_2 \ldots \rho_k \in S_n$ then

   a) $A + B$ is singular, when $\exists \rho_i, 1 \leq i \leq k$ has even length.

   b) $A + B$ is nonsingular, when $\forall \rho_j$, $1 \leq i \leq k$ has odd length.

5. If A and B are permutation matrices in $P(n)$ and $A^{-1}B$ corresponding to permutation $\sigma = \rho_1\rho_2 \ldots \rho_k \in S_n$, and $c_1, c_2 \in \mathbb{R} \setminus \{0\}$, $c_1 + c_2 \neq 0$, then

   a) $c_1A + c_2B$ is singular, when $\exists \rho_i, 1 \leq i \leq k$ has even length.

   b) $c_1A + c_2B$ is nonsingular, when $\forall \rho_j$, $1 \leq i \leq k$ has odd length.

6. Let B is a reflection matrix corresponded to $\sigma = \rho_1\rho_2 \ldots \rho_k$ for some $k \in \mathbb{N}$. Then there is at lest one ρ_j of even length where $1 \leq j \leq k$.

7. If A is rotation matrix and B is reflection matrix, then

$$\det(A + B) = \det A + \det B.$$